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Brane-antibrane systems at finite temperature and the entropy of black branes

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Abstract: We consider D-brane–anti-D-brane systems at $T > 0$. Starting at the closed string vacuum, we argue that a finite temperature leads to the reappearance of open string degrees of freedom. We also show that, at a sufficiently large temperature, the open string vacuum becomes stable. Building upon this observation and previous work by Horowitz, Maldacena and Strominger, we formulate a microscopic brane-antibrane model for the non-extremal black three-brane in ten dimensions (as well as for the black two- and five-branes in eleven dimensions). Under reasonable assumptions, and using known results from the AdS/CFT correspondence, the microscopic entropy agrees with the supergravity result up to a factor of $2^{\frac{p}{p+1}}$, with p the dimension of the brane. The negative specific heat and pressure of the black brane have a simple interpretation in terms of brane-antibrane annihilation. We also find in the model states resembling black holes and other lower-dimensional black branes.

Keywords: D-branes, Black Holes in String Theory.

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Contents

1. Introduction

Polchinski's identification of D-branes [1] as the microscopic counterparts of the RR-charged black brane solutions of type-II supergravities [2] opened the possibility to explain black brane entropy in terms of the open strings living on the D-branes. This possibility was realized by Strominger and Vafa [3], who were able to reproduce the exact entropy of certain extremal black holes through a microscopic state counting. The counting was soon extended to some near-extremal cases [4], and this was followed by a tremendous surge of activity [5], which led to the discovery of the AdS/CFT correspondence [6] as a remarkable by-product, and continues even today (see, e.g., the interesting recent works [7, 8, 9]).

Most of the examples where a successful microscopic description has been found involve extremal and near-extremal black branes. These systems have positive specific heat, and their entropy can be accounted for using a gas of finite temperature living on the D-branes. Black branes which are far from extremality, like the ordinary Schwarzschild black hole, are in this respect much more challenging. Consider for example the case of a neutral black three-brane of mass M wrapped on a torus of volume V. Its entropy is given by [5]

$$S = 2^{9/4} 5^{-5/4} \pi^{1/4} \sqrt{\kappa}\, M^{5/4} V^{-1/4} \,. \tag{1.1}$$

If one tries to interpret this as the entropy of a gas of particles with energy $E = M$ some well-known problems arise. First, since the power of E is larger than one, a simple thermodynamical calculation shows that the specific heat is negative. Second, by extensivity the exponent of V is forced to be negative, which implies that the pressure $p = T\,(\partial S/\partial V)_M$ is negative. Finally, when the black brane evaporates completely through Hawking radiation, all of its excitations disappear, meaning that the field theory which describes it microscopically should have a vacuum with no degrees of freedom other than (possibly) closed strings.

The last point suggests that the relevant field theory should perhaps display confinement: the black brane could then be associated with an unconfined vacuum, and the endpoint of the evaporation process would be identified with the confined vacuum, where flux tubes (closed strings) are the only allowed excitations. This could also explain the negative pressure: because of the energy difference between the confined and unconfined vacua, the system would gain energy by reducing its volume.

The starting point for the present work is the observation that these same properties are in fact possessed by D-brane–anti-D-brane systems. These systems were first analyzed in [10], and have been much studied of late [11]. The fact that the corresponding theory has a vacuum with only closed string excitations was conjectured by Sen [12]. The exact mechanism through which open string degrees of freedom disappear is still in debate; but a particularly promising proposal maintains precisely that the theory at the vacuum in question is confined [13], and that closed strings are to be understood as tubes of the confined flux [14]–[17].

In this paper we will argue that non-dilatonic black branes may be understood microscopically as collections of branes and antibranes. It had been noted already in [18] that the Bekenstein-Hawking entropy of the D1-D5 system arbitrarily far from extremality can be rewritten in a form suggestive of a microscopic model involving branes and antibranes (an observation which was extended to other cases in [19]). However, the understanding of D-$\overline{\mathrm{D}}$ systems was at that time insufficient to attempt a direct formulation of such a model — in particular, it would have been unclear why the branes and antibranes do not annihilate. As we will review in the next section, the situation in this regard has improved considerably in recent years. The significance of the tachyonic instability of the D-$\overline{\mathrm{D}}$ system is now understood, and the exact tachyon potential has been determined [20, 21]. Furthermore, it has been observed that on the worldvolume of a brane-antibrane system one can find solitons describing not only fundamental strings but also D-branes of lower dimensionality [12, 22, 15]. These results support the hope that the open string field theory associated with a space-filling brane-antibrane pair could give a non-perturbative definition of the full string theory. If this is correct, then it should certainly be possible to compute within this framework the entropy of black branes.

In the following we will examine these ideas in more detail. In particular, we will show that, under reasonable assumptions about the degrees of freedom (including their reduction by a factor of 3/4 due to strong-coupling [23, 24]), a model with branes, antibranes, and corresponding gases of open strings, can precisely reproduce the black brane entropy (1.1), up to a puzzling factor of $2^{3/4}$ which seems difficult to understand. We have already observed above that a D-$\overline{\mathrm{D}}$ system has the correct properties to explain the disappearance of the black brane's degrees of freedom after its evaporation, as well as its negative pressure. As we will see, this system can also account for the negative specific heat. The explanation is simply that, when energy is taken from the gas living on the branes, it is entropically favorable to annihilate some of the branes, which effectively increases the temperature of the gas. In other words, for a fixed number of branes the specific heat is of course positive, but the possibility of getting energy from brane annihilation (or tachyon condensation in Sen's language) makes the specific heat of the whole system negative.

The paper is structured as follows. We begin in section 2 by discussing generic features of the brane-antibrane system at finite temperature. We will argue that, starting at the closed string vacuum, a finite temperature leads to the (re)appearance of open string degrees of freedom, a phenomenon which might have implications for the nature of the Hagedorn transition. We will also show that, at a high enough temperature, the 'tachyon' field at the open string vacuum is no longer tachyonic, and so the D-$\overline{\mathrm{D}}$ system is stable. The analysis in later sections relies only on this last point, so readers interested mostly in black brane entropy considerations might wish to skip directly to section 2.3, where the point is made. In section 3 we will formulate a simple D-$\overline{\mathrm{D}}$ model for the black three-brane in ten dimensions (as well as the black two- and five-branes in eleven dimensions), and show that it correctly accounts for various properties of the supergravity solution. We discuss in detail the case of a Schwarzschild (neutral) black brane, and note that for the charged case supergravity curiously seems to suggest that the gases living on the branes and antibranes have the same energies, and therefore different temperatures. Section 4 demonstrates that our microscopic model can reproduce certain features of the Gregory-Laflamme instability [25]. Our conclusions are given in section 5.

Other attempts to give a microscopic description of Schwarzschild black holes have been made previously, most notably via the string/black hole correspondence [26, 27], and in the Matrix theory [28] context [29]–[32].

2. Brane-antibrane systems at finite temperature

The knowledge emerging from the vigorous investigation of unstable branes in recent years [11] has enhanced our understanding of the structure of string theory. This activity was initiated by Sen [12], who in particular conjectured that D-brane systems with a perturbative tachyonic excitation disappear completely via condensation of the associated scalar field (the 'tachyon'). At the end of this process the theory is defined in a new vacuum, whose excitations are described by closed strings alone.

The verification of this proposal requires tools capable of describing strings off-shell, and this has led to a resurgence of string field theories. Considerable progress has been made along three distinct but intersecting routes: cubic open string field theory [33] (see in particular the works [34, 22, 35]), noncommutative theories [37, 15] (for a review, see [38]), and background independent or boundary string field theory (BSFT)[1] [40, 41]. Each of these methods has illuminated different aspects of the process of tachyon condensation; together they have provided an impressive quantitative verification of Sen's proposal. These results can be viewed as an indication that string field theories truly constitute a non-perturbative formulation of string physics, and are thus a valuable complement to the formulations provided to us by Matrix theory [28] and Maldacena's duality [6].

By means of BSFT, in particular, it has been possible to determine the tachyon potential exactly[2] [43, 44, 21]. This potential has the property that its second derivative at the minimum (i.e., the mass of the tachyon,) is infinite, which is in agreement with our expectation that, at the end of the condensation process, all open string degrees of freedom are absent — the D-branes have disappeared. In this section we will argue, however, that the description of the closed string vacuum as a tachyon condensate has an interesting consequence: if we turn on a finite temperature, the open string degrees of freedom will reappear. Our main interest is the case of a brane-antibrane system in superstring theory, but the essential features are the same for unstable branes in bosonic or supersymmetric string theories.

2.1 A single brane-antibrane pair

For concreteness, we will carry out the analysis for a single D9-$\overline{\text{D9}}$ pair in type-IIB string theory. As always, the excitations of this D-brane system are described by open strings whose endpoints are anchored on the branes. The 9-9 and $\overline{9}$-$\overline{9}$ strings give rise to the usual (GSO-even) gauge fields and gauginos $\{A_\mu, \Lambda_\alpha\}$, $\{\overline{A}_\mu, \overline{\Lambda}_\alpha\}$ on the brane and the antibrane. The 9-$\overline{9}$ and $\overline{9}$-9 strings, on the other hand, yield GSO-odd states: a complex tachyon ϕ from the NS sector, and massless fermions $\Psi_\alpha, \overline{\Psi}_\alpha$ from the R sector. Since it originates from strings running between the brane and

[1] This is essentially a refinement of the old sigma-model approach. See [39] and references therein.

[2] Remarkably, it had been anticipated in [42, 20], where it was shown to possess all the desired features.

the antibrane, the tachyon is charged under the relative U(1) (i.e., $A^-_\mu \equiv A_\mu - \overline{A}_\mu$), but is neutral under the overall U(1) ($A^+_\mu \equiv A_\mu + \overline{A}_\mu$).

The exact potential for the tachyon takes the form [20, 21, 45]

$$V(\phi) = 2\tau_9 \exp[-2|t(\phi)|^2] \,, \qquad (2.1)$$

where τ_9 is the D9-brane tension, and t is the background tachyon field appearing in the worldsheet action, related to ϕ (the field with a standard kinetic term in the BSFT action) through the error function,

$$|\phi| = \sqrt{\frac{\pi}{2}}\mathrm{Erf}(|t|) \,. \qquad (2.2)$$

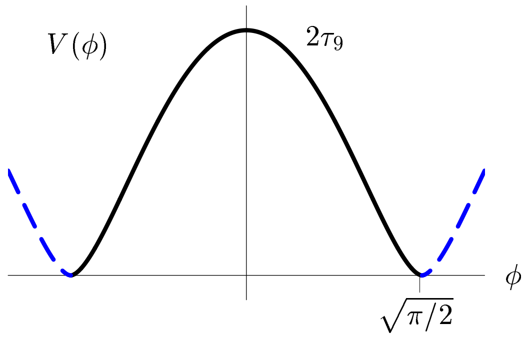

Figure 1: Tachyon potential for the brane-antibrane system. The physical range for the field is $|\phi| < \sqrt{\pi/2}$ ($0 \leq |t| < \infty$); the dotted lines outside this range have been drawn to aid the eye.

The potential (2.1) is of the 'Mexican hat' type (see figure 1): it has a maximum at $\phi = 0$ (the open string vacuum) and a minimum at $|\phi| = \sqrt{\pi/2}$ (the closed string vacuum). Close to the minimum it takes the form

$$V(\phi) = -4\tau_9 \left(\sqrt{\frac{\pi}{2}} - |\phi|\right)^2 \ln\left(\sqrt{\frac{\pi}{2}} - |\phi|\right) + \cdots \,. \qquad (2.3)$$

As noted before, the fact that the tachyon has an infinite mass at the closed string vacuum ($V''(\phi = \sqrt{\pi/2}) = +\infty$) is the concrete expression of its removal from the spectrum of the theory. This is in fact expected for all open string degrees of freedom, although the precise manner in which each mode disappears is not entirely clear. This is especially true for the gauge field A^+_μ: calculations in cubic string field theory suggest that it acquires a mass [35], whereas ordinary field theory intuition and BSFT suggest otherwise [46, 44] (see also [47, 16, 48, 49]). A promising mechanism for its ultimate removal from the theory is the proposal that the associated U(1) becomes confined [13]. This picture is attractive because, as a bonus, it allows one to understand closed strings at the $\phi = 0$ vacuum as collective excitations of the open string field: they are interpreted as electric flux tubes[3] of the confined U(1) [14, 17].

If we now consider this same system at a finite temperature, standard thermal field theory reasoning tells us what to expect: a small temperature should lead to an effective potential in which the location of the minimum has shifted away from $|\phi| = \sqrt{\pi/2}$. The physical reason for this is that moving towards $\phi = 0$ can be thermodynamically favorable: it costs energy, but it also reduces the mass of the tachyon and therefore increases the entropy of the tachyon gas. The optimal configuration is the one that minimizes the free energy of the system, and this will

[3]The idea that closed strings are electric flux tubes has been discussed independently of the confinement scenario in [15, 16].

vary with the temperature. For a large enough temperature the minimum could conceivably shift all the way to $\phi = 0$, in which case the open string vacuum would be stable.

When attempting to analyze this effect explicitly for the potential (2.1), one faces a number of difficulties. First, the field theory we are studying is non-renormalizable, not only because (2.1) describes an interacting scalar field in $9+1$ dimensions, but also because the complete action for the tachyon includes higher-derivative corrections (see [44, 50] and refs. therein). A related point is that the field theory (2.1) appears to be strongly coupled near the closed string vacuum, whereas string theory intuition would suggest that the coupling is $e^{-2|t|^2} g_s N$, which would mean that the system is weakly coupled for $|\phi| \to \sqrt{\pi/2}$ ($|t| \to \infty$). This puzzle was noted in [44] and has been further discussed in [17, 51]. The way to address these problems would be to work in the full string theory, including the effect of all the higher open string modes by performing a one-loop calculation in the BSFT language. For the zero-temperature case this has been attempted in various ways in [52, 51]; see especially [51] for a discussion of the issues involved.[4]

The need to resort to the full string theory highlights another qualification of our analysis: we will in what follows focus attention on the tachyon field alone, ignoring the contribution of all other open string modes to the free energy of the system. The rough picture we have in mind here is that as one approaches the closed string vacuum, the entire open string mass spectrum is shifted uniformly.[5] If the tachyon has the lowest mass, its contribution will be dominant as long as the temperature is low compared to the Hagedorn temperature. Summarizing, we hope that, despite its shortcomings, the minimalistic calculation that follows at least captures the essential qualitative features of the problem.

The free energy for the brane-antibrane + tachyon condensate + gas system at a finite temperature $T = \beta^{-1}$ is

$$F(\phi, \beta) = 2\tau_9 e^{-2|t|^2} + \frac{\Omega_8}{(2\pi)^9}\beta^{-10} \int_0^\infty dx\, x^8 \ln\left[1 - e^{-\sqrt{x^2+\beta^2 M^2}}\right], \qquad (2.4)$$

where Ω_8 is the volume of a unit 8-sphere and $M = \sqrt{V''(\phi)/4\tau_9} = \sqrt{|t|^2 - 1/2}$ is the mass of the particles in the gas (in $l_s = 1$ units). Assume now that the temperature is small enough that $e^{\beta M} \gg 1$ (note that in any case we are interested in temperatures lower than the Hagedorn temperature, i.e., $\beta \geq \beta_H = 2\sqrt{2}\pi$). Keeping only the first term in the expansion of the logarithm and carrying out the x integral, we are left

[4] The $T > 0$ case is examined in [53], which appeared while this paper was being written.

[5] It is amusing to note that this is what a naive application of BSFT on the *cylinder* would predict: a constant tachyon background $t = a$ in the worldsheet action simply multiplies the partition function by a factor of $e^{-a^2 l}$ (where l is the length of the boundary), which can be interpreted as a uniform mass-squared shift for all the modes. Given the relation (2.2) between t and ϕ, this shift in fact agrees with the tachyon mass shift inferred from $V''(\phi)$, $M^2 + 1/2 = t^2$.

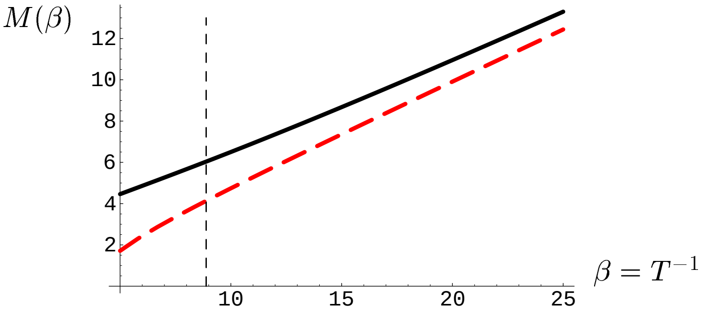

Figure 2: Mass of the open string tachyon in string units as a function of the inverse temperature, for $g_s N = 10^{-10}$ (solid line) and $g_s N = 1$ (dashed line), where N is the number of D9-$\overline{\mathrm{D9}}$ pairs. The vertical line indicates the Hagedorn temperature, $\beta_H = 2\sqrt{2}\pi$. See text for discussion.

with

$$F(\phi,\beta) \simeq 2\tau_9 e^{-2|t|^2} - 2(2\pi)^{-5}(M/\beta)^5 K_5(\beta M)\,. \tag{2.5}$$

Next, we would like to find the minimum of (2.5), i.e., the value of ϕ at which $\partial_\phi F(\phi,\beta) = 0$. It is most convenient to write this as an equation for the value of M at the minimum,

$$g_s(2\pi)^4(M/\beta)^4 K_4(\beta M)e^{2M^2+1} = 4\,. \tag{2.6}$$

To arrive at this equation we have made use of the fact that $\partial_x[x^\nu K_\nu(x)] = -x^\nu K_{\nu-1}(x)$, and we have introduced the string coupling constant g_s using the formula for the D9-brane tension, $\tau_9 = 1/g_s(2\pi)^9 l_s^{10}$. It can be seen from (2.6) that in the free string limit $g_s \to 0$ the finite-temperature effects disappear, and the minimum remains at $M = \infty$ ($\phi = 0$). The reason for this is simply that the D9 tension, which sets the scale of the zero-temperature potential (2.1), goes to infinity. We should therefore consider g_s to be small but finite.

The numerical solution to (2.6) is shown in figure 2. For $\beta, M \gg 1$ one can use the exponential approximation to the Bessel function to obtain the analytic expression

$$M(\beta) \simeq \frac{\beta}{2} + \frac{\ln\beta}{\beta} - \frac{1}{\beta}\left[\ln g_s + \frac{9}{2}\ln\pi - 2\ln 2 + 1\right], \tag{2.7}$$

which gives a good approximation to the exact solution for all β, g_s of interest, and makes the g_s-dependence manifest. As noted before, for any given β, $M(\beta) \to \infty$ as $g_s \to 0$.

It is seen from (2.7) and figure 2 that the mass M of the tachyon[6] is inversely proportional to the temperature, and that for reasonably small values of g_s it becomes

[6]Strictly speaking, the effective mass of the tachyon is not really M but the second derivative of (2.4), and one would ideally carry out a self-consistent calculation using this mass instead of M as an input to (2.4). However, as long as M is not too close to zero, the difference between these two masses is small, and so can be neglected to the accuracy of our analysis.

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of order one in string units at sub-Hagedorn temperatures (e.g., $M \sim 10$ at $\beta \sim 18$, which corresponds to $T \sim T_H/2$). This is then a clear indication that the open string degrees of freedom are no longer negligible at such temperatures.[7] As stated before, at high enough temperatures the minimum may shift all the way to $\phi = 0$, meaning that the open string vacuum is no longer tachyonic. The relevant temperature should be comparable to the energy needed to completely recreate the D9-$\overline{\text{D9}}$ pair, and therefore (for small g_s) larger than the Hagedorn temperature, which means it cannot be detected directly within our present framework. We will return to this issue in section 2.3, using a slightly different approach.

The preceding analysis has taught us that, since the tachyon field has the option of uncondensing only partially, one can have open strings (albeit of large mass) even if the energy density is lower than what is required to create a full D-$\overline{\text{D}}$ pair. Notice that, even though closed strings are certainly present as low energy excitations about the $|\phi| = \sqrt{\pi/2}$ vacuum, we have been able to ignore them in the foregoing analysis by working in the canonical ensemble, where the open and closed string gases simply have a common temperature. In the next subsection we will see that, in fact, it is more appropriate to use the microcanonical ensemble, where a given total energy U has to be be distributed among the components of the system in such a way as to maximize the entropy. For small U, this means that the energy will go entirely to the closed string gas, the open strings becoming relevant only at a rather large value of U. Note, however, that one is free to consider a state with a large number N of brane-antibrane pairs, as we will do in more detail in the next subsection. In the regime $g_s \ll g_s N \ll 1$, a state with D-$\overline{\text{D}}$ pairs and open strings decays only very slowly into closed strings, and can be considered metastable as long as it is stable with respect to the open string interactions, governed by $g_s N$. It would be interesting to investigate this state further using the boundary state formalism to obtain the supergravity fields that it generates (however one has to take into account the fact that the system is at finite temperature [54]).

Even though states of this sort have lower entropy than a gas of closed strings with the same energy, they could conceivably play a role in certain physical phenomena. One such phenomenon might be the Hagedorn transition, about which we would now like to make some speculative remarks.

Recall first that the Hagedorn temperature for closed strings is 'non-limiting': it can be reached by supplying a finite amount of energy. Open strings, on the other hand, have a limiting Hagedorn temperature (for detailed discussion on this and related issues, see, e.g., [55, 56]). This fundamental distinction makes the closed string

[7]Of course, the fact that the theory is strongly-coupled near the closed string vacuum means that the free mass $M^2 \sim V''$ can receive large corrections [17]. But even if we do not interpret M as the tachyon mass, a low value of M implies that the minimum has shifted significantly. Note also that, away from $|\phi| = \sqrt{\pi/2}$, the coupling can be reduced by taking g_s to be small enough.

case rather mysterious,[8] and throughout the years several authors have speculated about the possible nature of the phase transition associated with the non-limiting behavior (see, e.g., [57], or the recent works [58] and refs. therein).

If one considers string theory models of hadrons, then the Hagedorn transition is naturally associated with the deconfinement transition. In that context strings are flux tubes of the color electric field, which disappear above the critical temperature, since the flux can spread. If instead one considers closed strings to be fundamental objects, this picture does not seem relevant. However, as discussed recently, in the context of unstable brane systems closed strings can in fact be described as flux tubes of the open string gauge field, which has been argued to be confined in the closed string vacuum [13, 17]. If the analogy can be pushed further, the Hagedorn transition could perhaps be related with the spread of these flux tubes, which is only possible if D-branes appear. This would imply that for finite values of the string coupling constant, the closed string spectrum changes drastically as one approaches the Hagedorn temperature, and the closed strings ultimately disintegrate into open strings, which cannot then be heated above the Hagedorn temperature.

Remember also that the usual analysis of string thermodynamics reveals that as one approaches the Hagedorn temperature the energy goes into a single highly excited string, which looks like a random walk that fills the entire volume available. If this long string is built from electric flux, then it seems likely that the state can be seen as a bubble where electric flux is unconfined.

Unfortunately, the simple-minded calculations of the present section are not enough to pursue these ideas further, even if they suggest that a relation between the Hagedorn transition and the emergence of open string degrees of freedom is possible. Closely related remarks have been made in [56].

2.2 Multiple brane-antibrane pairs

How would the discussion in the previous subsection change for $N > 1$ D9-$\overline{\mathrm{D9}}$ pairs? Strings running between the branes and the antibranes give rise to N^2 complex tachyon fields which can be assembled into an $N \times N$ matrix $t_{a\bar{c}}$, $1 \leq a, c \leq N$. The potential is [21]

$$V(\phi) = 2\tau_9 \,\mathrm{Tr} \exp[-2|t(\phi)|^2] \,. \tag{2.8}$$

The matrices $(t\,t^\dagger)_{ac}$ and $(t^\dagger t)_{\bar{a}\bar{c}}$ can be simultaneously diagonalized by forming appropriate linear combinations of the branes and (independently) the antibranes, so one can restrict attention to the case where only the diagonal tachyons condense. The potential (2.8) is minimized by

$$t_{a\bar{c}} = t\delta_{ac}, \quad t \to \infty \qquad \Longrightarrow \qquad V = 0\,, \tag{2.9}$$

[8] We are grateful to Bo Sundborg for emphasizing this point to us.

which leads to a shift in the mass of all N^2 tachyons. For our purposes, the result is to increase the first and second terms in (2.5) by respective factors of N and N^2, reflecting the energy of the branes and the number of tachyon species:

$$F(\phi,\beta) = 2N\tau_9 e^{-2|t|^2} - 2(2\pi)^{-5}N^2(M/\beta)^5 K_5(\beta M)\,. \qquad (2.10)$$

The net effect is simply to replace g_s with g_sN in eqs. (2.6) and (2.7), a fact which has already been taken into account in the caption of figure 2.

Having noted the possibility for an arbitrary number N of brane-antibrane pairs to reemerge from the vacuum, it becomes clear that the value of N is not arbitrary and should be determined thermodynamically. The N-dependence displayed by (2.10), and the relative minus sign between the two terms, imply that a plot of the free energy as a function of N has the shape of an inverted parabola, and is therefore minimized at $N = \infty$ for all $\beta < \infty$. We thus conclude that, in the canonical ensemble, the minimum (2.7) is unstable: the system can lower its free energy by increasing N (with M decreasing in the process), driving the coupling constant g_sN into the non-perturbative regime. The closed string vacuum at finite temperature is of course already known to be unstable due to black hole formation, and in section 4 we will see that these two types of instabilities are in fact related (see [56] for related discussions). At the same time, it is clear that the system is not unstable in a microcanonical analysis: given a finite amount of energy, the entropy vanishes if we use it all up to create D-$\overline{\text{D}}$ pairs, or if we create no pairs at all, so there is an intermediate value of N which maximizes the entropy.

Our main goal thus far has been to understand the phenomenon of reappearance of open strings in the closed string vacuum at finite temperature, but it is also interesting to think in these terms about the reverse process. Consider N D9-$\overline{\text{D9}}$ pairs at the open string vacuum. As we know, they are unstable and will decay by tachyon condensation. In order for energy to be conserved in the process, the energy gained from sliding down the tachyon potential must be used to produce strings, closed or open. We can ensure that all of the energy goes into open strings by taking $g_s \to 0$ while keeping g_sN fixed: emission of closed strings by the branes is then forbidden. The endpoint of the condensation process will then consist of an open string gas on N partially condensed branes. The final tachyon expectation value will lie close to $|\phi| = \sqrt{\pi/2}$ for $g_sN \ll 1$, and it will move towards $\phi = 0$ as g_sN is increased. For large enough g_sN, then, it is conceivable that the tachyon will not condense at all. We will now examine this possibility in more detail.

2.3 Critical temperature

For use in the next section, we will carry out this part of the analysis for a D3-$\overline{\text{D3}}$ system. We would like to determine the conditions under which the open string vacuum, $t = \phi = 0$, is a minimum of the effective tachyon potential. When this happens, the 'tachyon' will no longer be tachyonic, so (at sub-Hagedorn temperatures) the massless fields will make the most important contribution to the free energy. The tachyon gas in our previous calculations must consequently be replaced by a gas of gluons (+ transverse scalars + superpartners). If one starts at $\phi = 0$, then sliding down the tachyon potential (2.8) lowers the energy of the system, but it also gives mass to the relative U(N) gauge fields, and so decreases the entropy of the gas. We are interested in establishing which of these effects dominates.

The free energy for the brane-antibrane + tachyon condensate + gas system at temperature $T = \beta^{-1}$ is given by the obvious modification of (2.4),

$$F(\phi,\beta) = 2\tau_3 \,\mathrm{Tr}\, e^{-2|t|^2} + \frac{\Omega_2}{(2\pi)^3} c N^2 \beta^{-4} \int_0^\infty dx\, x^2 \ln\left[\frac{1-e^{-\sqrt{x^2+\beta^2 m^2}}}{1+e^{-\sqrt{x^2+\beta^2 m^2}}}\right], \qquad (2.11)$$

where $\tau_3 = 1/g_s(2\pi)^3 l_s^4$, $m \sim |\phi|$ is the mass given to the gluons by the Higgs effect, and the numerical constant $c = 8$ for the relevant 8 bosonic + 8 fermionic degrees of freedom (gauge field + transverse scalars + superpartners). Starting at $\phi = 0$ and letting a single diagonal tachyon condense[9] by an amount $\delta\phi$ gives mass to N out of the N^2 species of particles in the gas, and so changes (2.11) by

$$\delta F = -4\tau_3(\delta\phi)^2 + \frac{1}{2} N \beta^{-2} (\delta m)^2\,, \qquad (2.12)$$

which is positive for large enough temperature. Disregarding the numerical constants, we thus learn that for

$$T \geq \frac{T_H}{\sqrt{g_s N}}\,. \qquad (2.13)$$

the open string vacuum is a minimum of the free energy (equivalently, a maximum of the entropy for fixed total energy[10]), and the brane-antibrane pairs do not annihilate.

To arrive at (2.12) we have considered the mass that the relative gauge fields (and scalars) acquire due to their coupling to the tachyon, but we can equivalently phrase the result in the opposite direction: the second term of (2.12) represents a mass term for ϕ due to a thermal expectation value for the relative gauge fields, $\langle A^- A^- \rangle_T \sim N T^2$, corresponding to a mass $m_\phi \sim \sqrt{g_s N} T$.

It is important to note that the regime (2.13) lies in the physically accessible sub-Hagedorn region only for non-perturbative values of the coupling, $g_s N > 1$, where we would expect the system to have a dual supergravity description. In the following sections we will use this insight to model black branes in supergravity in terms of brane-antibrane systems.

[9] Letting all diagonal tachyons condense (see the discussion following (2.8)) would yield a factor of N in the first term of (2.12), but it would also give mass to all N^2 species of particles, leaving condition (2.13) unchanged.

[10] As we have seen in the previous subsection, a microcanonical analysis is preferable because in the presence of a heat reservoir the system is unstable towards creation of an infinite number of brane-antibrane pairs.

3. Entropy of black branes

In the previous section we found that at a large enough temperature there is a stable state of a D-$\overline{\mathrm{D}}$ system where a finite number of brane-antibrane pairs are uncondensed. In order for this temperature regime to be physically accessible the system must be strongly-coupled, $g_s N > 1$. It is thus natural to expect the uncondensed D-$\overline{\mathrm{D}}$ state to have an alternative supergravity description, which could only be a black brane at the same temperature. In fact, it is known that in many cases [18, 19] the energy and entropy of black branes admit an interpretation in terms of branes and antibranes. In this section we explore this idea more closely for the case of black three-branes in type-IIB string theory and black two- and five-branes in M-theory. These non-dilatonic cases have been analyzed in their near-extremal regime in [23, 59], and play an important role in the AdS/CFT correspondence [6]. The regime far from extremality has been considered from the point of view of Matrix theory [29]–[32] and by means of the string/black hole correspondence [26, 27] but, to the best of our knowledge, has not been discussed in the context of brane-antibrane models. We will carry out most of the discussion for the best-understood case of D3-branes, leaving the M2- and M5-brane cases for section 3.4.

3.1 Brane-antibrane model for black three-branes

The model that we consider is the low-energy theory on the worldvolume of a system of N D3-branes and $\bar{N}$ anti-D3-branes. From the gauge theory point of view this means that the original $\mathrm{U}(\infty) \times \mathrm{U}(\infty)$ gauge group of the full theory is broken down (by the expectation value of the tachyon) to $\mathrm{U}(N) \times \mathrm{U}(\bar{N})$. Since the numbers N and $\bar{N}$ of uncondensed branes and antibranes can vary, the actual values are chosen to maximize the total entropy of the system for fixed charge and mass.

The temperature is assumed (and later confirmed) to satisfy $1/\sqrt{g_s N} \ll T \ll 1$ (and similarly with $\bar{N}$) in string units. As seen in (2.13), the first inequality is required for stability; the second one allows us to ignore the massive open string modes. For the above temperature range to exist we must have $g_s N \gg 1$, $g_s \bar{N} \gg 1$, so we are necessarily in the strong-coupling regime. We will take $g_s \ll 1$ to suppress closed string loops. Under these conditions, when a brane-antibrane pair annihilates the energy goes to the gas of open strings on the remaining branes and antibranes, rather than being emitted as closed strings (Hawking radiation), since the latter process is disfavored for small g_s.

Since we are trying to formulate a microscopic model for a supergravity solution, the restriction to strong coupling was of course expected. Notice, however, that the situation is rather peculiar in that there is no weakly-coupled stable vacuum: if $g_s N < 1$, then for any physical temperature the open string vacuum is unstable and the D-$\overline{\mathrm{D}}$ pairs will condense, driving the system towards the strongly-coupled closed string vacuum. This means that, unlike other cases, here we do not have the option of studying the system at weak-coupling and then extrapolating to strong-coupling.

In the absence of a weakly-coupled regime, and knowing that the theory is not supersymmetric, the best one can do is to use plausibility arguments to determine the entropy. If the result turns out to agree with supergravity, perhaps the only conclusion should be that supergravity predicts a reasonable behavior for the system at strong coupling. More optimistically, one could view an eventual agreement as evidence for the validity of the microscopic model. With these caveats in mind, we now proceed to compute the entropy.

The theory on the branes is $(3+1)$-dimensional $\mathcal{N}=4$ super-Yang-Mills (SYM) with gauge group $\mathrm{U}(N)$. The theory on the antibranes is also $\mathcal{N}=4$ SYM, with gauge group $\mathrm{U}(\bar{N})$, although the supersymmetry generators of the brane and antibrane theories are different. At weak coupling these theories would give $8N^2+8\bar{N}^2$ massless bosons, $2N^2+2\bar{N}^2$ from the gauge fields $(A^\mu_{ac},\bar{A}^\mu_{\bar{a}\bar{c}})$ and $6N^2+6\bar{N}^2$ from the scalars $(\Phi^i_{ac},\,\bar{\Phi}^i_{\bar{a}\bar{c}})$, plus the same number of massless fermions $(\Lambda^\alpha_{ac},\,\bar{\Lambda}^\alpha_{\bar{a}\bar{c}})$. In addition, the theories are coupled together by the fields arising from brane-antibrane strings, which transform in the bifundamental of $\mathrm{U}(N)\times\mathrm{U}(\bar{N})$. These are $N\bar{N}$ complex tachyons $(\phi_{a\bar{c}})$ and $16N\bar{N}$ massless fermions $(\Psi^\alpha_{a\bar{c}})$ coming from the Ramond sector.

At strong-coupling the counting is modified. A qualitative approach (which as we will see does not give a full answer) is to observe that the cubic and quartic couplings give mass to some of the fields when others have expectation values [4, 27]. If the mass is larger than the temperature, then these degrees of freedom are frozen and should not be counted.

Let us remind ourselves how this works in the case where there are only D-branes. The scalars for example get a mass from the term

$$g_s\,\mathrm{Tr}[\Phi^i,\Phi^j]^2 = 2g_s\sum_{a,b,c,d=1}^{N}\left(\Phi^i_{ab}\Phi^i_{cd}\right)\left(\Phi^j_{bc}\Phi^j_{da}-\delta_{bc}\Phi^j_{de}\Phi^j_{ea}\right). \tag{3.1}$$

On dimensional grounds $\langle(\Phi^j_{ab})^2\rangle\sim T^2$, so at large N the last term gives a mass of order $\sqrt{g_sN}\,T$ to all the fields. The same is valid for the gauge bosons and the fermions.[11] In the regime $g_sN\gg 1$ this mass is much larger than the temperature, so in principle these degrees of freedom should not be excited. However, if they are not excited, then they do not get a large mass. For example, if only the diagonal fields (corresponding to strings going from one brane to itself) acquire expectation values, then the off-diagonal fields are given a mass of order $m\sim T$, which is not large enough for them to be suppressed. This contradiction means that the naive argument does not provide a definite answer, but it does suggest that the situation is intermediate between these two extremes. At present, our limited understanding

[11] In the case of the fermions the relevant coupling is $\sqrt{g_s}\Phi^i_{ab}\bar{\Lambda}_{bc}\Gamma^i\Lambda_{ca}$. Since $\langle\mathrm{Tr}\,\Phi\rangle=0$ and $\langle\mathrm{Tr}\,\Phi^2\rangle\sim N^2\langle(\Phi_{ab})^2\rangle\sim N^2T^2$, the eigenvalues φ^i of Φ^i satisfy $|\varphi^i|\sim\sqrt{N}T$.

of this case comes from the AdS/CFT correspondence [6], which predicts that the entropy at $g_s \ll 1 \ll g_s N$ is the same as that of a free field theory with only $6N^2$ bosons and the same number of fermions, instead of the naive $8N^2$ [23, 24].

Going back to the case of branes and antibranes, the situation is similar. We can apply the above argument to show that, if $\mathcal{O}(N^2)$ fields are excited on the branes and $\mathcal{O}(\bar{N}^2)$ on the antibranes, then the tachyons $\phi_{a\bar{c}}$ and the fermions $\Psi_{a\bar{c}}$ acquire a mass of order $\sqrt{g_s N}\,T$, which is in fact what we obtained (for the tachyon) in section 2.3. If such is the case, the fact that (for $g_s N \gg 1$) the temperature is much lower than their mass means that these fields are not excited, and the theory on the branes decouples from the theory on the antibranes. For these two decoupled theories, the AdS/CFT correspondence predicts that indeed $\mathcal{O}(N^2)$ and $\mathcal{O}(\bar{N}^2)$ fields are excited, respectively, making the result self-consistent.

The upshot of this long discussion is that the energy, entropy and charge of our microscopic system should be given by

$$M_{\rm FT} = (N+\bar{N})\tau_3 V + a\frac{\pi^2}{16}N^2 V T^4 + a\frac{\pi^2}{16}\bar{N}^2 V \bar{T}^4 \,, \tag{3.2}$$

$$S_{\rm FT} = a\frac{\pi^2}{12}N^2 V T^3 + a\frac{\pi^2}{12}\bar{N}^2 \bar{T}^3 \,, \tag{3.3}$$

$$Q = N - \bar{N} \,, \tag{3.4}$$

where $\tau_3 = 1/(2\pi)^3 g_s l_s^4$ denotes the tension of the D3-branes. The numerical constant $a = 8$ if we consider 8 free bosons and 8 free fermions, but $a = 6$ if we use the strong-coupling input from the AdS/CFT correspondence [23, 24], as we will do from now on. We have taken the theory to be compactified on a torus of volume V. The above formulas would be expected to hold only for $T > V^{-1/3}$, but in fact they are valid even for much smaller temperatures because the branes can be multiply-wrapped, which effectively means the gases live in a much larger volume [60, 27]. Finally, notice that since the theories are decoupled, the gas on the branes and the gas on the antibranes could a priori have different temperatures, $T \neq \bar{T}$.

The energy (3.2) includes a contribution from the brane/antibrane tension (or equivalently, from the tachyon potential) and another from the gases. These two contributions can be distinguished by considering the full energy-momentum tensor, because the tachyon potential contributes with a minus sign to the space-space components of $T_{\mu\nu}$:

$$T_{00} = (N+\bar{N})\tau_3 + a\frac{\pi^2}{16}N^2 T^4 + a\frac{\pi^2}{16}\bar{N}^2 \bar{T}^4 \,, \tag{3.5}$$

$$T_{ij} = \left[-(N+\bar{N})\tau_3 + a\frac{\pi^2}{48}N^2 T^4 + a\frac{\pi^2}{48}\bar{N}^2 \bar{T}^4 \right] \delta_{ij} \,, \tag{3.6}$$

where $i,j = 1,2,3$ and we have used the fact that the trace of $T_{\mu\nu}$ is zero for a gas of massless particles. So, for example, one can separate the energy due to the brane tension by computing $T = \eta^{\mu\nu} T_{\mu\nu}$, which receives no contribution from the gases. Expressions (3.5) and (3.6) are useful to us because the same logic can be applied in the supergravity side: $T_{\mu\nu}$ can be computed using the asymptotic value of the black brane metric, and then both contributions can be distinguished also in that case.

The last step in the formulation of the microscopic model is to maximize $S_{\rm FT}$ with respect to N and $\bar{N}$, keeping $M_{\rm FT}$ and Q fixed. In the next two subsections we will do this and then compare the result with supergravity.

3.2 Comparison with supergravity: neutral case

We consider here the neutral case, $Q = N - \bar{N} = 0$, leaving the more general case to the next subsection. Since $N = \bar{N}$, by symmetry we expect the temperatures to be equal, $T = \bar{T}$. Thus, the energy and entropy are given by

$$M_{\rm FT} = 2N\tau_3 V + a\frac{\pi^2}{8} N^2 V T^4 \,, \tag{3.7}$$

$$S_{\rm FT} = a\frac{\pi^2}{6} N^2 V T^3 \,. \tag{3.8}$$

As discussed in the previous subsection, including strong-coupling effects we expect $a = 6$. If one is not interested in the numerical coefficient then a can be taken as an unknown constant, and the correct functional form of the entropy should follow just from the fact there are $\mathcal{O}(N^2)$ massless degrees of freedom.

The value of N is determined by maximizing the entropy at fixed $M_{\rm FT}$. From the above relations we obtain

$$S_{\rm FT} = a^{1/4}\frac{2^{5/4}}{3}\sqrt{\pi}\,\sqrt{N}\,V^{1/4}\left(M_{\rm FT} - 2NV\tau_3\right)^{3/4} \,, \tag{3.9}$$

which is maximized by

$$N = \frac{1}{5}\frac{M_{\rm FT}}{\tau_3 V} \,. \tag{3.10}$$

This is easily seen to imply that the energy contained in the gases is 3/2 of the total tension of the branes and antibranes, a prediction which as is shown below agrees with supergravity. Plugging the equilibrium value (3.10) back into (3.9) we obtain the entropy-energy relation

$$S_{\rm FT} = a^{1/4} 2^{5/4} 3^{-1/4} 5^{-5/4} \pi^{1/4} \sqrt{\kappa}\, V^{-1/4} M_{\rm FT}^{5/4} \,, \tag{3.11}$$

where we have expressed the D3-brane tension in terms of the gravitational coupling constant, $\tau_3 = \sqrt{\pi}/\kappa$. In addition, we find that the temperature is $T \sim (g_s N)^{-1/4}$, which as required satisfies $1/\sqrt{g_s N} \ll T \ll 1$.

To compare with supergravity, we recall that a neutral black three-brane with Schwarzschild radius r_0 has mass and entropy [5]

$$M_{\rm SG} = \frac{5}{2}\frac{\pi^3}{\kappa^2} r_0^4 V \,,$$

$$S_{\rm SG} = \frac{2\pi^4}{\kappa^2} r_0^5 V \,, \tag{3.12}$$

which implies that

$$S_{\rm SG} = 2^{9/4} 5^{-5/4} \pi^{1/4} \sqrt{\kappa}\, V^{-1/4} M_{\rm SG}^{5/4} \,. \tag{3.13}$$

Identifying $M_{\rm SG} = M_{\rm FT}$, we see that the functional form of the supergravity and field theory entropies agree. The numerical coefficient would agree for $a = 48$, but as we have seen the AdS/CFT correspondence implies that $a = 6$, meaning that the field theory entropy is a factor of $2^{3/4}$ too small, $S_{\rm SG} = 2^{3/4} S_{\rm FT}$. Equivalently, one can say that the supergravity entropy behaves as if the gases carried twice the available energy.

A similar factor was found already in [18] for the D1/D5 system far away from extremality. The authors observed that considering separate gases to account for the entropy results in an energy 4 times larger than the correct value. We will see in section 3.4 that the same factor appears in the M2- and M5-brane cases after one takes into account the AdS/CFT prediction for the strong-coupling behavior of the worldvolume theory.

Another interesting check is to consider the energy-momentum tensor. From the asymptotic value of the gravitational field one finds that [31] (see also the appendix): $T_{ij} = -\frac{1}{5} T_{00} \delta_{ij}$. Putting $T_{00} = \mathcal{T} + E$ and $T_{ij} = (-\mathcal{T} + \frac{1}{3} E)\delta_{ij}$, where $\mathcal{T}$ is the contribution from the brane/antibrane tension and E from the gases (see the previous subsection), one obtains $E = \frac{3}{2}\mathcal{T}$, in agreement with the field theory prediction.

Before moving on to the charged case, let us discuss an interesting issue that appears already here. Since we reproduce the black brane entropy, it is clear that the specific heat of our system is negative. To understand why, let us consider how Hawking evaporation proceeds in this model, and check that the temperature increases when energy is radiated. When a closed string is emitted, energy is taken from the open string gas, so a priori the temperature should decrease. However, we have found that, in equilibrium, the energy in the gas is 3/2 the tension of the D-$\overline{\rm D}$ pairs. This means that, when the gas has lost enough energy so as to match 3/2 the tension of $N - 1$ pairs, it will be entropically favorable for one pair to annihilate, giving energy to the gas and increasing its temperature. This is repeated again and again, effectively increasing the temperature on average as the mass of the system decreases. The process will continue until $g_s N \sim 1$, where the gas reaches the Hagedorn temperature ($T \sim 1$ in string units) and the model (3.2)–(3.4), based only on the massless open string modes, ceases to be valid. At this point we would expect all of the available energy to go into a highly-excited (open or closed) long string, so the brane-antibrane model makes contact with the string/black hole correspondence [26, 27]. Indeed, $g_s N \sim 1$ is precisely the correspondence point, where the curvature of the black brane at the horizon is of order the string scale, and the Bekenstein-Hawking entropy is known to match the entropy of a long string at the Hagedorn temperature [27].

3.3 Comparison with supergravity: general case

We start by considering the supergravity expressions, following a procedure similar to [18]. The energy, entropy and charge of a non-extremal three-brane are given by [5]

$$M_{SG} = \frac{\pi^3}{\kappa^2} r_0^4 V \left(\cosh 2\alpha + \frac{3}{2}\right), \tag{3.14}$$

$$S_{SG} = \frac{2\pi^4}{\kappa^2} r_0^5 V \cosh\alpha\,, \tag{3.15}$$

$$Q_{SG} = \frac{\pi^{\frac{5}{2}}}{\kappa} r_0^4 \sinh 2\alpha\,. \tag{3.16}$$

As discussed before, it is interesting to consider not only the mass but also the other components of the energy-momentum tensor. As shown in the appendix, it turns out to be

$$T_{00} = \frac{\pi^3}{\kappa^2} r_0^4 \left(\cosh 2\alpha + \frac{3}{2}\right), \tag{3.17}$$

$$T_{ij} = \left[\frac{\pi^3}{\kappa^2} r_0^4 \left(-\cosh 2\alpha + \frac{1}{2}\right)\right] \delta_{ij}\,. \tag{3.18}$$

Comparing (3.14), (3.16) and (3.18) with (3.2), (3.4) and (3.6) uniquely determines

$$N = \frac{\pi^{5/2}}{2\kappa} r_0^4 e^{2\alpha}\,, \qquad \bar{N} = \frac{\pi^{5/2}}{2\kappa} r_0^4 e^{-2\alpha}\,. \tag{3.19}$$

The energy of the gases is then identified with

$$E = \frac{3}{2}\frac{\pi^3}{\kappa^2} V r_0^4\,, \tag{3.20}$$

in terms of which the entropy can be written as

$$S_{\rm SG} = 2^{5/4} 3^{-3/4} \pi^{1/2} V^{1/4} E^{3/4} \left(\sqrt{N} + \sqrt{\bar{N}}\right) \tag{3.21}$$

From (3.2) and (3.3) with $a = 6$, we see that this is the correct entropy for a gas of particles on the N branes plus another gas on the $\bar{N}$ antibranes, both with the same energy E. However, since the total energy available for the gases is E, in the field theory model we have to assign an energy $E/2$ to each of them, resulting in a mismatch in the entropy which is exactly the same as found for the neutral case in the previous subsection: $S_{\rm SG} = 2^{3/4} S_{\rm FT}$. Under the condition that the energies in both gases are the same, one can check that the expressions (3.19) are the ones that maximize the entropy for fixed charge and mass.

The fact that the energy densities (or equivalently the pressures) of the two gases are the same implies that their temperatures T,$\bar{T}$ are different. They are related through

$$\frac{2}{T_{\rm FT}} = \frac{1}{T} + \frac{1}{\bar{T}}\,, \tag{3.22}$$

where $T_{\rm FT}$ is the temperature defined as $T_{\rm FT}^{-1} = (\partial S/\partial M)_Q$. Due to the discrepancy in the numerical coefficient of the entropy, $T_{\rm FT}$ is a factor of $2^{3/4}$ smaller than the Hawking temperature of the black brane, $T_h = 1/\pi r_o \cosh\alpha$. An expression of the same form as (3.22) appeared in the analysis of the D1/D5 system, relating the Hawking temperature to the temperatures of the left- and right-moving modes in the microscopic description [4]. Remarkably, in that case the two distinct temperatures could be recognized also in the supergravity side, by computing the greybody factor [61]. It would be interesting to see whether the same can be done here, examining the absorption probabilities for the non-extremal three-brane. For the near extremal case the absorption probability was computed in [62] and also in [63] where a much more precise analysis was done. It would be interesting to extend the results to the region far from extremality.

As we emphasized before, since the theories on the branes and antibranes are decoupled, there is nothing to prevent us from postulating that the temperatures of the corresponding gases are different. However, it is not clear to us why supergravity seems to require that their energies be the same.[12] It is conceivable that states in which the gases have different energies (or equivalently, different pressures) are allowed microscopically. If so, they could correspond to other supergravity backgrounds (perhaps like the ones found in [64]), or they might not even have a supergravity description. A clue in this respect might be the observation that, under this condition, the fluctuations in the transverse positions of the branes and the antibranes turn out to be equal, and of the order of the Schwarzschild radius. This follows from the fact that such fluctuations are measured by the eigenvalues of the $N \times N$ scalar field matrix Φ^i_{ac}, $(i = 1, \ldots, 6)$. As noted in subsection 3.1, the eigenvalues are of order NT^2 for the branes and $\bar{N}\bar{T}^2$ for the antibranes, with agreement if both gases have the same energy density $(N^2T^4 = \bar{N}^2\bar{T}^4)$.

The equality of the two energies implies that, as one approaches the extremal limit $M = Q\mathcal{T}_3 V$ (i.e., $\alpha \to \infty$ with $r_0 \propto e^{-\alpha/2}$), the temperature of the gas on the antibranes grows without bounds. On the other hand, since the model is based on massless open string modes, it is expected to be valid only if both T and $\bar{T}$ are substantially lower than the Hagedorn temperature $T_H \sim 1/l_s$. From the microscopic perspective, we know that the Hagedorn temperature is in this case limiting (see, e.g., [55, 56]), and we expect that as $\bar{T} \to T_H$, the energy available to the gas on the antibranes goes to a highly excited long string, whose contribution to the entropy is however negligible compared to the gas on the branes.

In the near-extremal region, it is known that the black brane entropy can be precisely reproduced using a system without antibranes [23], so as one approaches

[12] A similar correlation between the various components of the system was observed in [18]. It is intriguing to note the similarity between these correlations (and their accompanying numerical discrepancies) and certain aspects of the field theory model for eternal AdS black holes that was very recently formulated in [7].

extremality one would intuitively expect a transition to this class of states. It is easy to see that the entropies of these two microscopic descriptions cross when $M \simeq 6Q\tau_3 V$, which indeed suggests a transition, with the brane-antibrane system being the preferred one further away from extremality. However, it is hard to see how one could retain the agreement with supergravity in the entire parameter space: the brane-antibrane model gives the exact dependence of the black brane entropy on M and Q, but gives a numerical value which is always a factor of $2^{3/4}$ too small, whereas the model of [23] is in accord with supergravity in the near-extremal region, but deviates significantly from it already at the presumed transition point. To summarize, at least from the microscopic point of view there appears to be a discontinuity in taking the near-extremal limit, but the issue clearly deserves futher study.

3.4 Two- and five-branes in eleven dimensions

The calculation of the entropy can easily be generalized to the case of M2- and M5-branes. For the neutral case, our microscopic model is again of the form

$$M_{\mathrm{FT}} = 2N\tau_p V + aN^c V T^{p+1}\,, \tag{3.23}$$

with the entropy given by

$$S_{\mathrm{FT}} = \frac{p+1}{p} aN^c V T^p\,, \tag{3.24}$$

where c and a are numerical constants which depend on the type of brane. They follow from the near-extremal case [59] or equivalently, from the AdS/CFT correspondence. Proceeding in the same way as in the case of the D3-brane one finds

$$S_{\mathrm{FT}} = 2^{-\frac{c}{p+1}} a^{\frac{1}{p+1}} \tau_p^{-\frac{c}{p+1}} (p+1) p^{-\frac{1}{p+1}} (c+p)^{-\frac{p+c}{p+1}} c^{\frac{c}{p+1}} V^{\frac{1-c}{p+1}} M^{\frac{c+p}{p+1}}\,. \tag{3.25}$$

In the case of the M5-brane the tension is given by

$$\tau_5 = 2^{-1/3}\pi^{1/3}\kappa^{-4/3}\,, \tag{3.26}$$

and the calculations for the near-extremal case suggest that

$$a = 2^7 3^{-7} 5\pi^3\,, \qquad c = 3\,. \tag{3.27}$$

This gives an entropy

$$S_{\mathrm{FT}} = 2^{-13/6} 3^{1/3} \pi^{1/3} \kappa^{2/3} V^{-1/3} M^{4/3}\,, \tag{3.28}$$

which has the right dependence on the mass and volume as compared with the supergravity result [5], but is a factor $2^{5/6}$ too small.

For the M2-brane the tension is given by

$$\tau_2 = 2^{4/3}\pi^{2/3}\kappa^{-2/3}\,, \tag{3.29}$$

and the near-extremal case suggests that

$$a = 2^{11/2} 3^{-4} \pi^2 \,, \qquad c = \frac{3}{2} \,, \tag{3.30}$$

so one finds

$$S_{\rm FT} = 2^{3/2} 3^{1/6} 7^{-7/6} \pi^{1/3} \kappa^{1/3} V^{-1/6} M^{7/6} \,, \tag{3.31}$$

which again has the right behavior but this time is a factor $2^{2/3}$ too small. It is intriguing that the numerical factor can be corrected in the same way for all three models: the $2^{p/(p+1)}$ discrepancy means that in the three cases the supergravity entropy behaves as if the gas carried twice the available energy.

The charged case is again similar to the D3-brane described in the previous subsection. The supergravity formulas can be found in [5]. The black five-brane solution in eleven dimensions has

$$\begin{aligned}
M_{\rm SG} &= \frac{2\pi^2}{\kappa^2} r_0^3 V \left(\cosh 2\alpha + \frac{5}{3}\right) , &(3.32)\\
T_{ij} &= \frac{2\pi^2}{\kappa^2} r_0^3 \left(-\cosh 2\alpha + \frac{1}{3}\right) \delta_{ij} \,, &(3.33)\\
S_{\rm SG} &= \frac{16\pi^3}{3\kappa^2} r_0^4 V \cosh\alpha \,, &(3.34)\\
Q &= \frac{2^{\frac{3}{2}}\pi^2}{\kappa^{2/3}} r_0^3 \sinh 2\alpha \,, &(3.35)
\end{aligned}$$

while for the black two-brane one has

$$\begin{aligned}
M_{\rm SG} &= \frac{\pi^4}{2\kappa^2} r_0^6 V \left(\cosh 2\alpha + \frac{4}{3}\right) , &(3.36)\\
T_{ij} &= \frac{\pi^4}{2\kappa^2} r_0^6 \left(-\cosh 2\alpha + \frac{2}{3}\right) \delta_{ij} \,, &(3.37)\\
S_{\rm SG} &= \frac{2\pi^5}{3\kappa^2} r_0^7 V \cosh\alpha \,, &(3.38)\\
Q &= \frac{\pi^4}{\sqrt{2}\kappa^{4/3}} r_0^6 \sinh 2\alpha \,. &(3.39)
\end{aligned}$$

It is easy to see that these formulas can again be interpreted in terms of a brane-antibrane model with

$$\begin{aligned}
M_{\rm FT} &= (N+\bar{N})\tau_p V + aN^c V T^{p+1} + aN^c V \bar{T}^{p+1} \,, &(3.40)\\
T_{ij} &= \left[-(N+\bar{N})\tau_p V + \frac{1}{p}\left(aN^c V T^{p+1} + aN^c V \bar{T}^{p+1}\right)\right] \delta_{ij} \,, &(3.41)\\
S_{\rm FT} &= \frac{p+1}{p}\left(aN^c V T^p + aN^c V \bar{T}^p\right) , &(3.42)\\
Q &= N - \bar{N} \,, &(3.43)
\end{aligned}$$

and that the inferred values of N and $\bar{N}$ correctly maximize the entropy for a given mass and charge. Just like in the D3-brane case, one finds here that, to match the supergravity expressions, the energy densities of both gases have to be the same. The constant c turns out to be the same as in the near-extremal case [59], but a differs by a factor of 2^p as before.

4. Instabilities of black branes

As shown by Gregory and Laflamme [25], black p-branes are generally unstable, and tend to split into lower-dimensional branes. This can be seen thermodynamically, by showing that lower-dimensional branes of the same mass have larger entropy, or dynamically, by studying metric perturbations. The best studied example is the black string, which is believed to collapse into a series of black holes (see however, the recent paper [65]). If the black string is charged, one expects the black holes to be threaded by an extremal (or near-extremal) black string carrying the charge. For higher-dimensional branes one has similar expectations. That the thermodynamical and the dynamical instabilities are related is not obvious. Recently it has been conjectured and checked in several examples that the classically unstable mode appears when the entropy is not a local minimum [66]. This is another remarkable example of the deep connection between gravity and thermodynamics.

An important property of the Gregory-Laflamme transition is that it does not occur if the black brane is wrapped on a torus of sufficiently small size, as can be seen by examining the expressions for the entropy. For simplicity, we will restrict attention to the case of a brane with no charge. The entropy of a neutral black p-brane in D dimensions is given by

$$S_p = 2\pi\Omega_{n+1}^{-\frac{1}{n}} \left(\frac{2}{n+1}\right)^{\frac{n+1}{n}} \kappa^{\frac{2}{n}} V_p^{-\frac{1}{n}} M^{\frac{n+1}{n}} \,, \tag{4.1}$$

where $n = D - p - 3$, Ω_{n+1} is the volume of a unit $(n+1)$-sphere, and V_p is the volume of the p-dimensional torus on which the brane is wrapped. If the radii of the torus are small enough the black p-brane is, according to the above reasoning, stable, while if one of the radii, R, is too large the black p-brane will tend to collapse in the corresponding direction forming a black $(p-1)$-brane. For this to occur the lower-dimensional brane should be the configuration with the highest entropy. Ignoring numerical factors, we find from setting $S_p \sim S_{p-1}$ that the critical value of R is given by

$$R \sim \left(\frac{\kappa^2 M}{V_{p-1}}\right)^{\frac{1}{n+1}} \,,$$

which coincides with the Schwarzschild radius (or equivalently, the inverse Hawking temperature) of the $(p-1)$-brane. If the conjectured equivalence between the dynamical and thermodynamical instabilities is correct, we conclude that the p-brane

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is stable only if *all* radii are smaller than this critical value, i.e.,

$$R_i < l_P (M l_P)^{\frac{1}{D-3}} \qquad \forall\, i = 1, \ldots, p\,, \tag{4.2}$$

where $l_P \sim \kappa^{2/(D-2)}$ is the D-dimensional Planck length.

Can the properties of the Gregory-Laflamme transition be understood in terms of our brane-antibrane construction? It is clear that, since we have the correct formulas for the entropy, the thermodynamical instability is exactly the same. From this point of view, the instability means that if we enclose the system in a finite volume and hold its total energy fixed, then its entropy will increase when the volume decreases. In other words, the system prefers not to occupy the entire volume available, and it can accomplish this by annihilating the D-$\overline{\mathrm{D}}$ pairs in one region of space to create additional pairs in another. This is not in contradiction with the fact that, as we discovered in section 2.3, at the temperatures of interest the 'tachyon' has a positive mass-squared, and the open string vacuum is stable. The analysis there was restricted to the case of a constant tachyon, whereas here we are considering a space-dependent perturbation which can be described as a local variation in the number of branes and antibranes. Such perturbation can be unstable even if the constant one is not. It would be interesting to pursue this issue further and find the dynamical equations of the perturbation to obtain the unstable mode analogous to the one found in supergravity by Gregory and Laflamme [25]. Here, however, we limit ourselves to a simpler but nevertheless interesting question. We have noted that the entropy increases as we reduce the volume, but if we reduce the volume too much, the temperature might not be high enough to excite the gas, and the entropy would in fact decrease. It is then conceivable that the microscopic model stabilizes itself at some finite volume. Since we keep the total energy fixed, and not the temperature, it is not clear a priori whether this happens or not. We will analyze this issue in the next subsections, and in the process obtain a simple model for a ten-dimensional black hole in terms of branes and antibranes. The fact that certain aspects of the Gregory-Laflamme instability could be explained in terms of a D-brane model was noted already in [27].

4.1 Collapse into a black hole

We consider now the case where all of the cycles of the torus violate the bound (4.2), and so, for the purposes of this section, can be considered to be infinite. The brane is then unstable, and is expected to reduce to a black hole. Our microscopic model tells us that the mass of the neutral black brane can be written as

$$M = 2\tau_p N L^p + E\,, \tag{4.3}$$

where E is the energy of the gas and L is the size of the brane system, which can vary due to tachyon condensation. Both E and the entropy S are proportional to N^c, where $c = 2, 3/2, 3$ for the case of D3-, M2-, and M5-branes, respectively.

If we regard $M = M(N, L, T)$ and $S = S(N, L, T)$, then maximizing the entropy while holding the mass fixed means enforcing the two conditions

$$\left(\frac{\partial S}{\partial N}\right)_{M,L} = \frac{\partial S}{\partial N} - \frac{1}{T}\frac{\partial M}{\partial N} = 0\,, \tag{4.4}$$

$$\left(\frac{\partial S}{\partial L}\right)_{M,N} = \frac{\partial S}{\partial L} - \frac{1}{T}\frac{\partial M}{\partial L} = 0\,, \tag{4.5}$$

where the second one is the condition of vanishing pressure. The first equality in (4.4) and (4.5) follows from the chain rule and the fact that (4.3) implies

$$\left(\frac{\partial S}{\partial M}\right)_{N,L} = \left(\frac{\partial S}{\partial E}\right)_{N,L} = \frac{1}{T}\,. \tag{4.6}$$

With the help of (4.3) and $S, E \propto N^c$, we can rewrite (4.4) as

$$cS - \frac{M}{T} + \frac{1-c}{T}E = 0\,. \tag{4.7}$$

By dimensional analysis, $S \sim S\,(LT)$, and $E \sim f\,(LT)\,/L$ for some function f. Using this and $M - E \propto L^p$, together with (4.6) and (4.7), we can rewrite the first equality in (4.5) as

$$\left(\frac{\partial S}{\partial L}\right)_{M,N} = \frac{1}{TL}\left((p+1)\,E - pM\right). \tag{4.8}$$

Now, for a large p-brane the equation of state for the gas is $S = \frac{p+1}{p}\frac{E}{T}$. With the help of (4.7) and (4.8), this implies that the system has negative pressure,

$$\left(\frac{\partial S}{\partial L}\right)_{M,N} = -\frac{(c-1)\,p}{p+c}\frac{M}{TL} < 0\,, \tag{4.9}$$

with the consequence that the branes will tend to contract. That is, eq. (4.5) can never be satisfied: as we knew already, there is no local minimum. We would now like to consider how finite-size effects change the form of the entropy. The partition function for the gas (including the bosonic and fermionic degrees of freedom) is given by

$$\ln Z \sim N^c \sum_{\vec{n}\in\mathbb{Z}^p} \ln\left(\frac{e^{|\vec{n}|/LT}+1}{e^{|\vec{n}|/LT}-1}\right), \tag{4.10}$$

When the size becomes smaller than $1/T$, the partition function is dominated by the $|\vec{n}| = 1$ term,

$$\ln Z \sim N^c e^{-1/TL}\,, \tag{4.11}$$

from which one easily derives

$$E \sim \frac{N^c}{L} e^{-1/TL} \tag{4.12}$$

and

$$S = LE + \frac{1}{T}E\,. \tag{4.13}$$

The crucial question is now whether this change will be sufficient to stop the contraction. For this to occur the pressure (4.8) must vanish, i.e.,

$$E = \frac{p}{p+1}M\,. \tag{4.14}$$

Using (4.7) and (4.13) one finds that this will indeed happen at a critical size

$$L = \frac{1}{pc}\frac{1}{T}\,. \tag{4.15}$$

To summarize, we have shown that a black brane wrapped on a large torus is unstable, and will tend to break and contract until it reaches the size quoted above. What are the characteristics of the final object? Using (4.14) and (4.15) in (4.13) we find that $c(p+1)S = (pc+1)M/T$, and with the help of (4.6) we can infer from this that

$$S \sim M^{\frac{c(p+1)}{pc+1}}\,. \tag{4.16}$$

More precisely, if we use the values appropriate for the D3-brane case, $p = 3$ and $c = 2$, we find

$$S \sim (Ml_{P,10})^{8/7}\,, \tag{4.17}$$

which is the expected entropy for a ten-dimensional black hole! The M2-brane, with $p = 2$ and $c = 3/2$, and the M5-brane, with $p = 5$ and $c = 3$, both give the appropriate entropy for an eleven dimensional black hole:

$$S \sim (Ml_{P,11})^{9/8}\,. \tag{4.18}$$

Notice also that the size of the final object is commensurate with the Schwarzschild radius r_0 of the corresponding black hole: one finds (in Planck units) $L \sim M^{1/7}$ for the D3-brane case and $L \sim M^{1/8}$ for the M2- and M5-brane cases. By (4.15), this is equivalent to saying that the temperature of the gas agrees with the Hawking temperature of the black hole. It is intriguing that in the microscopic model $r = r_0$ has a definite meaning as the place where the system ends; it is difficult to imagine how an infalling observer could fail to notice this type of 'horizon', as must be the case for a large macroscopic black hole.

All in all we have obtained a simple model for the ten-dimensional black hole. It is perhaps worth noticing that the expression (4.17) can be writen as $S \sim N^2$, that is, the entropy is proportional to the square of the number of branes-antibrane pairs, and depends on the volume and temperature only through N. This comes from the fact that there are N^2 modes with fixed spatial dependence ($|\vec{n}| = 1$). This is similar to what happens with a 'small' black hole in global AdS space. From there we know that a ten-dimensional black hole can be described by constant matrices of size $N \times N$ (see [67]), which is quite similar to what we have here. It would be interesting to further explore the properties of the present model, to see if they agree with the expectations from supergravity.

We have so far shown that a p-brane wrapped on a large torus is unstable, and that the end product will be a stable black hole, in complete agreement with the supergravity expectations. What if the size R of the torus is small? We then need to show two things: that the standard formula for the entropy of the brane continues to hold for the small tori, and that there is no instability towards a collapse into something else. The crucial observation in this context is that the branes can connect with one another to form a single multiply-wrapped brane [60, 27, 29]. Effectively this means fewer degrees of freedom living in a larger volume. In the D3-brane case, for instance, we find N degrees of freedom, rather than N^2, living in the volume NV, rather than V. For large V this implies that the formula for the entropy remains exactly the same whether the branes are multiply-wrapped or not. For small V, however, there is an important difference: in the multiply-wrapped case the entropy formula continues to be of the standard form down to much smaller size. The reason is of course that the effective volume is really NV, and as a consequence finite-size effects are delayed. This is crucial since otherwise we would not have been able to explain the entropy of the black brane for small tori, which is precisely where the system should be stable. Furthermore, ignoring the Gregory-Laflamme instability, the supergravity expression for the entropy of a p-brane does not show any discontinuity at the the critical size.

Still, we must make sure that the instability that we observed for large tori is no longer present when $R < 1/T$. The key is again the multiple wrapping. If the tachyon condenses leaving a bubble of uncondensed tachyons of size $L < R$, it is clear that within this bubble we cannot have any multiply-wrapped branes. Since $L < 1/T$, finite-size effects are important, and as we have seen above these would yield the smaller entropy (4.13). From this we conclude that, on a small torus, the (multiply-wrapped) brane-antibrane system is stable, in agreement with the expectations for its supergravity counterpart. A subtlety is that it might be possible for the tachyon to condense in such a way as to create a hole on the branes and antibranes, without disrupting the fact that they are multiply-wrapped. To probe the stability of the system against such local perturbations would require a more detailed analysis of the dynamics than what we consider in this paper.

4.2 Collapse into lower-dimensional branes

What if we let only some of the cycles of the p-dimensional torus on which the system lives be larger than the bound (4.2)? Can our model reproduce the entropies of the resulting lower-dimensional black branes? To be more precise, let us say that $p - \tilde{p}$ of the radii of the $\mathbf{T}^p$ exceed the stability bound. The D-$\overline{\mathrm{D}}$ pairs will then partially annihilate, and the size L of the system along these directions will decrease. The entropy increases if we decrease L, but as we show below this ceases to be true when $L \sim 1/T$. Since our model accounts for the entropy of the initial black branes, the analysis between eqs. (4.1) and (4.2) predicts that in this case the system will have the entropy-energy relation of a $\tilde{p}$-brane, up to a numerical factor of order unity. This can be verified explicitly by extremizing the entropy with respect to N in our model, obtaining

$$S \sim M^{\frac{c(p+1)-\tilde{p}(c-1)}{(p-\tilde{p})(c-1)+p+1}}. \tag{4.19}$$

In order for this to match the general formula (4.1) we must have that

$$\frac{c\,(p+1)-\tilde{p}\,(c-1)}{(p-\tilde{p})\,(c-1)+p+1}=\frac{D-\tilde{p}-2}{D-\tilde{p}-3}, \tag{4.20}$$

or equivalently,

$$\frac{p+1}{c-1}=D-3-p, \tag{4.21}$$

which is true *precisely* for the values of p and c appropriate for the D3-, M2- and M5-brane cases.

Now we have to check that the entropy really ceases to increase if L is of order $1/T$. The brane-antibrane system has topology $\mathbf{T}^{\tilde{p}} \times \mathbf{I}^{p-\tilde{p}}$, where $\mathbf{I}$ is an interval of a certain length L. Outside of this interval the tachyon is fully condensed and we have the closed string vacuum, so the masses of all open string fields tend to infinity. Because of this, the mass-terms in the action force all fields to go to zero at the end points of $\mathbf{I}$. This implies that there is no constant mode on $\mathbf{I}$, and exciting any mode requires at least an energy $1/L$. If $T < 1/L$ no modes are fully excited, and the entropy decreases in a manner similar to what we saw in section 4.1, which is what we wanted to show.

The possibility of describing lower-dimensional black branes in terms of higher-dimensional ones is clearly an extension to the non-extremal case of Sen's identification of D-branes as lumps in the tachyon field [12, 22]. It is intriguing to note that in the case of eleven dimensions we have found two different ways to obtain the lower-dimensional black branes. For instance, the black two-brane can either be viewed as pairs of M2's and $\overline{\mathrm{M2}}$'s, or as pairs of M5's and $\overline{\mathrm{M5}}$'s where the tachyon has condensed along three directions.

5. Conclusions

In this paper we have studied brane-antibrane systems at finite temperature. It is intuitively obvious that a large enough temperature will lead to the creation of D-$\overline{\mathrm{D}}$ pairs, and as we have seen in section 2, the recent results in tachyon condensation can be used to make this idea more precise. The field theory description allows in fact the creation of partially-condensed brane-antibrane pairs, for which the expectation value of the tachyon lies at neither the open string nor the closed string vacuum. As we have seen, in the canonical ensemble the system is unstable towards creation of an infinite number of brane-antibrane pairs, so it should really be studied in the microcanonical ensemble, where only a finite amount of energy is available. It is then favorable to put all of the energy into closed strings, rather than use it to create branes. However, in the regime $g_s \ll 1$ states with $N \gg 1$ D-$\overline{\mathrm{D}}$ pairs and open strings can be long-lived, because the emission of closed strings is suppressed. If we keep $g_s N$ fixed, then the open strings have non-trivial dynamics. We argued that the value of $g_s N$ and the energy density determine the nature of the stable brane-antibrane state (which is really meta-stable if we take into account the closed strings).

For $g_s N \ll 1$, there is a stable state with the tachyon partially condensed, that we described in sections 2.1 and 2.2. This state contains partially annihilated D-$\overline{\mathrm{D}}$ pairs and massive open strings living on them. When the temperature is of the order of $1/l_s$ (with a weak dependence on g_s), the masses are of order $\sim 1/l_s$. This fact might be important when describing closed strings at high temperatures, as for example in the investigation of the possible nature of the Hagedorn transition, or near a black hole horizon.

For $g_s N \gg 1$, on the other hand, we showed in section 2.3 that the 'tachyon' field is no longer tachyonic at the open string vacuum if the temperature satisfies $T > 1/\sqrt{g_s N}$. The D-$\overline{\mathrm{D}}$ pairs then do not annihilate, and their excitations include massless open strings. The number N of brane-antibrane pairs in this stable state cannot be chosen arbitrarily; it is determined thermodynamically by maximizing the entropy keeping the total energy fixed. The entropy of the system is clearly zero if we do not create any brane-antibrane pairs (no degrees of freedom) or if we create the maximum number possible (largest number of degrees of freedom but no energy left to excite them), so the entropy will be maximized by an intermediate value of N such that the mass of the branes and the energy of the open string gas are of the same order. This state exists only in the non-perturbative regime $g_s N > 1$, and it is therefore natural to associate it with a black brane solution of supergravity at the same temperature. Going back to the issue of closed string emission, in the supergravity description that would correspond to Hawking radiation, which is indeed suppressed for $g_s \ll 1$.

In section 3 we examined more closely the identification between the stable brane-antibrane state and a black brane in supergravity. Building upon [18], we formulated an explicit microscopic model for the black three-brane in type-IIB theory, and also for the two- and five-brane in M-theory. The model involved a stack of branes, a stack of antibranes, and a gas of massless open string modes on each stack. We argued that, at low energies, the theory on the branes is decoupled from the theory on the antibranes. We were then able to use the AdS/CFT correspondence to compute the entropy of each strongly-coupled system separately, and then add them together.

The resulting entropy agreed with the supergravity result up to a puzzling factor of $2^{\frac{p}{p+1}}$, with p the dimension of the corresponding brane (in all cases this can be interpreted as a factor of 2 in the energy). Since the AdS/CFT correspondence uses supergravity (in the near-extremal region), the result might appear to be merely a consistency check, but the fact is that we are describing a very different regime (including the Schwarzschild case), and a different situation, since the number of branes can change. The agreement we find is, at least to us, a very unexpected property of the supergravity expressions.

A puzzling fact is that, when the number of branes differs from the number of antibranes, to reproduce the supergravity result one has to assume that the energy densities (or equivalently, the pressures) of the two gases are the same, implying that their temperatures are different. Since we argued that the theories are decoupled, it is certainly not out of the question that the temperatures could be different, but we lack an understanding of why precisely the states with equal energies on the two gases correspond to the supergravity solution.

Since we obtain the correct entropy-energy relation, it is clear that other properties of the black branes are reproduced. In particular, the specific heat of our microscopic system is negative. When the system loses energy, the energy density of the gas decreases, which would seem to imply that its temperature decreases. However, the gas is no longer capable of sustaining the same number of branes. A pair annihilates and the resulting energy is added to the gas, increasing the temperature.

Another important property of the supergravity solution is the fact that the black brane entropy increases as we decrease the volume. This is the thermodynamical formulation of the Gregory-Laflamme instability. If the black three-brane, for instance, lives on a very large torus, then it has lower entropy than a ten-dimensional black hole, so the latter is the preferred configuration. In section 4 we showed that in our microscopic model it is also convenient to reduce the size of the system, but only until it is of the order of the inverse temperature. Beyond that point the temperature is no longer large enough to create excitations on the branes, so the entropy in fact decreases. The radius that maximizes the entropy was found to yield an entropy-energy relation for the system which is precisely that of a black hole in ten dimensions, as expected from the supergravity side. The entropy is proportional to N^2 and is carried by the components of the matrix fields, rather than by their spatial dependence. This is important since it means that the gas on the branes does not probe distances smaller than the Schwarzschild radius. Contrary to what is expected for a normal gas, increasing the total energy of our system does not help to probe shorter distances (instead, the energy is used up in creating brane-antibrane pairs), in analogy with the behavior of a black hole.

The situation is different if the system lives on a sufficiently small torus. Since the branes can be multiply-wrapped, the large-volume formulas are valid even if the temperature is smaller than the inverse size of the torus. Brane-antibrane annihilation would destroy the multiple wrapping and thus lower the entropy, so the microscopic system is in this case stable, again in agreement with the black brane in supergravity.

To summarize, the field theory model we have studied in this paper possesses several appealing features which are in close correspondence with the properties of black branes and black holes in supergravity. Beyond the immediate task of questioning its assumptions and attempting to resolve the numerical discrepancy in the entropy, there are many other aspects that remain quite obscure, in particular the usual questions about black hole formation, complementarity, etc. By analogy with the AdS/CFT correspondence, it would also be interesting to see in what sense the near-horizon geometry of the black brane (i.e., Rindler space) is encoded in the microscopic description. Most important of all, the model should be tested in ways other than just computing the entropy.

For instance, similarly to [61] it will be interesting to compare the greybody factors with the field theory predictions. Also, one should be able to explicitly find the macroscopic perturbation of the system that is the equivalent of the Gregory-Laflamme unstable mode in supergravity. We believe that these and other aspects of the model merit further study.

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A. The energy-momentum tensor and the asymptotic metric

In the text we found it useful to know all the components of the energy-momentum tensor rather than just the energy. In order to do that we follow the procedure of [31], where more details can be found. We start by considering the D3-brane solution

$$ds^2 = h^{-1/2}(-f dt^2 + dy_{||}^2) + h^{1/2}(f^{-1}dr^2 + r^2 d\Omega_5^2)\,, \tag{A.1}$$

$$h = 1 + \sinh^2(\alpha)\frac{r_0^4}{r^4}\,, \tag{A.2}$$

$$f = 1 - \frac{r_0^4}{r^4}\,. \tag{A.3}$$

The solution can be written in isotropic coordinates by means of a change of radial variables

$$2\rho^4 = r^4 - \frac{1}{2}r_0^4 + \sqrt{r^8 - r_0^4 r^4}\,, \tag{A.4}$$

$$ds^2 = h^{-1/2}(-f\,dt^2 + dy_{\|}^2) + \frac{r(\rho)^2}{\rho^2}(d\rho^2 + \rho^2 d\Omega_5^2)\,. \tag{A.5}$$

For $\rho \to \infty$ we can write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with

$$h_{00} \simeq \left(1 + \frac{1}{2}\sinh^2(\alpha)\right)\frac{r_0^4}{\rho^4}\,, \tag{A.6}$$

$$h_{ij} \simeq -\frac{1}{2}\sinh^2(\alpha)\frac{r_0^4}{\rho^4}\delta_{ij}\,, \tag{A.7}$$

$$h_{ab} \simeq \left(\frac{1}{4} + \frac{1}{2}\sinh^2(\alpha)\right)\frac{r_0^4}{\rho^4}\delta_{ab}\,, \tag{A.8}$$

where i,j are spatial indices parallel to the brane and a,b denote the perpendicular directions. It is easy to see that $h_{\mu\nu}$ satisfies the harmonic gauge condition

$$\partial_\lambda h^\lambda_\mu - \frac{1}{2}\partial_\mu h = 0\,, \qquad h = \eta^{\mu\nu}h_{\mu\nu}\,, \tag{A.9}$$

which in turns simplifies the linear Einstein equations to

$$\partial_\lambda\partial^\lambda\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = 16\pi G T_{\mu\nu}\,. \tag{A.10}$$

Inserting the asymptotic values of $h_{\mu\nu}$ we get

$$T_{00} = \frac{\pi^3}{\kappa^2}r_0^4\left(\cosh(2\alpha) + \frac{3}{2}\right)\delta(x_\perp)\,, \tag{A.11}$$

$$T_{ij} = \left[\frac{\pi^3}{\kappa^2}r_0^4\left(-\cosh(2\alpha) + \frac{1}{2}\right)\right]\delta_{ij}\delta(x_\perp)\,, \tag{A.12}$$

$$T_{ab} = 0\,, \tag{A.13}$$

as used in the text. The delta-function in the transverse coordinates disappears after integrating over a small volume around the brane to get the $3+1$ dimensional $T_{\mu\nu}$.

The same calculation can be done for the M2- and M5-branes, to obtain the results used in the text.

References

[1] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [`hep-th/9510017`].

[2] G.T. Horowitz and A. Strominger, *Black strings and p-branes*, *Nucl. Phys.* **B 360** (1991) 197.

[3] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [`hep-th/9601029`].

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[4] C.G. Callan Jr. and J.M. Maldacena, *D-brane approach to black hole quantum mechanics*, *Nucl. Phys.* **B 472** (1996) 591 [hep-th/9602043];
G.T. Horowitz and A. Strominger, *Counting states of near-extremal black holes*, *Phys. Rev. Lett.* **77** (1996) 2368 [hep-th/9602051].

[5] Reviews and extensive lists of references can be found in:
J.M. Maldacena, *Black holes in string theory*, hep-th/9607235;
M. Cvetič, *Properties of black holes in toroidally compactified string theory*, *Nucl. Phys.* **56B** *(Proc. Suppl.)* (1997) 1 [hep-th/9701152];
G.T. Horowitz, *Quantum states of black holes*, gr-qc/9704072;
D. Youm, *Black holes and solitons in string theory*, *Phys. Rep.* **316** (1999) 1 [hep-th/9710046];
A.W. Peet, *The bekenstein formula and string theory (N-brane theory)*, *Class. and Quant. Grav.* **15** (1998) 3291 [hep-th/9712253];
A.W. Peet, *TASI lectures on black holes in string theory*, hep-th/0008241;
E.J. Martinec, *Black holes and the phases of brane thermodynamics*, hep-th/9909049;
S.R. Das and S.D. Mathur, *The quantum physics of black holes: results from string theory*, *Ann. Rev. Nucl. Part. Sci.* **50** (2000) 153 [gr-qc/0105063].

[6] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[7] J.M. Maldacena, *Eternal black holes in anti-de Sitter*, hep-th/0106112.

[8] C.V. Johnson and R.C. Myers, *The enhancon, black holes and the second law*, hep-th/0105159;
N.R. Constable, *The entropy of 4d black holes and the enhancon*, hep-th/0106038.

[9] R. Emparan and E. Teo, *Macroscopic and microscopic description of black diholes*, *Nucl. Phys.* **B 610** (2001) 190 [hep-th/0104206];
D. Kabat, G. Lifschytz and D.A. Lowe, *Black hole entropy from non-perturbative gauge theory*, hep-th/0105171.

[10] T. Banks and L. Susskind, *Brane-antibrane forces*, hep-th/9511194.

[11] Reviews on the subject can be found in: A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207;
A. Lerda and R. Russo, *Stable non-BPS states in string theory: a pedagogical review*, *Int. J. Mod. Phys.* **A 15** (2000) 771 [hep-th/9905006];
J.H. Schwarz, *TASI lectures on non-BPS D-brane systems*, hep-th/9908144;
M.R. Gaberdiel, *Lectures on non-BPS Dirichlet branes*, *Class. and Quant. Grav.* **17** (2000) 3483 [hep-th/0005029];
K. Ohmori, *A review on tachyon condensation in open string field theories*, hep-th/0102085.

[12] A. Sen, *Stable non-BPS bound states of BPS D-branes*, *J. High Energy Phys.* **08** (1998) 010 [hep-th/9805019]; *Tachyon condensation on the brane antibrane system*, *J.*

High Energy Phys. **08** (1998) 012 [hep-th/9805170]; *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105]; *Universality of the tachyon potential*, *J. High Energy Phys.* **12** (1999) 027 [hep-th/9911116].

[13] P. Yi, *Membranes from five-branes and fundamental strings from Dp-branes*, *Nucl. Phys.* **B 550** (1999) 214 [hep-th/9901159];
O. Bergman, K. Hori and P. Yi, *Confinement on the brane*, *Nucl. Phys.* **B 580** (2000) 289 [hep-th/0002223].

[14] G.W. Gibbons, K. Hori and P. Yi, *String fluid from unstable D-branes*, *Nucl. Phys.* **B 596** (2001) 136 [hep-th/0009061].

[15] J.A. Harvey, P. Kraus, F. Larsen and E.J. Martinec, *D-branes and strings as non-commutative solitons*, *J. High Energy Phys.* **07** (2000) 042 [hep-th/0005031];
F. Larsen, *Fundamental strings as noncommutative solitons*, *Int. J. Mod. Phys.* **A 16** (2001) 650 [hep-th/0010181].

[16] A. Sen, *Fundamental strings in open string theory at the tachyonic vacuum*, hep-th/0010240.

[17] M. Kleban, A.E. Lawrence and S.H. Shenker, *Closed strings from nothing*, *Phys. Rev.* **D 64** (2001) 066002 [hep-th/0012081].

[18] G.T. Horowitz, J.M. Maldacena and A. Strominger, *Nonextremal black hole microstates and U-duality*, *Phys. Lett.* **B 383** (1996) 151 [hep-th/9603109].

[19] M. Cvetič and D. Youm, *Entropy of non-extreme charged rotating black holes in string theory*, *Phys. Rev.* **D 54** (1996) 2612 [hep-th/9603147];
G.T. Horowitz, D.A. Lowe and J.M. Maldacena, *Statistical entropy of nonextremal four-dimensional black holes and U-duality*, *Phys. Rev. Lett.* **77** (1996) 430 [hep-th/9603195];
R. Kallosh and A. Rajaraman, *Brane-anti-brane democracy*, *Phys. Rev.* **D 54** (1996) 6381 [hep-th/9604193];
J.-G. Zhou, H.J. W. Muller-Kirsten, J.Q. Liang and F. Zimmerschied, *M-branes, anti-M-branes and non-extremal black holes*, *Nucl. Phys.* **B 487** (1997) 155 [hep-th/9611146];
M.S. Costa and M.J. Perry, *Landau degeneracy and black hole entropy*, *Nucl. Phys.* **B 520** (1998) 205 [hep-th/9712026].

[20] J.A. Minahan and B. Zwiebach, *Effective tachyon dynamics in superstring theory*, *J. High Energy Phys.* **03** (2001) 038 [hep-th/0009246].

[21] D. Kutasov, M. Mariño and G.W. Moore, *Remarks on tachyon condensation in superstring field theory*, hep-th/0010108.

[22] J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *J. High Energy Phys.* **04** (2000) 012 [hep-th/0002117];

J. Kluson, *D-branes in type-IIA and type-IIB theories from tachyon condensation*, hep-th/0001123.

[23] S.S. Gubser, I.R. Klebanov and A.W. Peet, *Entropy and temperature of black 3-branes*, *Phys. Rev.* **D 54** (1996) 3915 [hep-th/9602135].

[24] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *Coupling constant dependence in the thermodynamics of $N = 4$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 534** (1998) 202 [hep-th/9805156].

[25] R. Gregory and R. Laflamme, *Black strings and p-branes are unstable*, *Phys. Rev. Lett.* **70** (1993) 2837 [hep-th/9301052];
R. Gregory and R. Laflamme, *The instability of charged black strings and p-branes*, *Nucl. Phys.* **B 428** (1994) 399 [hep-th/9404071].

[26] M.J. Bowick, L. Smolin and L.C. Wijewardhana, *Does string theory solve the puzzles of black hole evaporation?*, *Gen. Rel. Grav.* **19** (1987) 113;
L. Susskind, *Some speculations about black hole entropy in string theory*, hep-th/9309145;
E. Halyo, A. Rajaraman and L. Susskind, *Braneless black holes*, *Phys. Lett.* **B 392** (1997) 319 [hep-th/9605112];
E. Halyo, B. Kol, A. Rajaraman and L. Susskind, *Counting Schwarzschild and charged black holes*, *Phys. Lett.* **B 401** (1997) 15 [hep-th/9609075];
G.T. Horowitz and J. Polchinski, *Self gravitating fundamental strings*, *Phys. Rev.* **D 57** (1998) 2557 [hep-th/9707170];
T. Damour, *Quantum strings and black holes*, gr-qc/0104080.

[27] G.T. Horowitz and J. Polchinski, *A correspondence principle for black holes and strings*, *Phys. Rev.* **D 55** (1997) 6189 [hep-th/9612146].

[28] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[29] T. Banks, W. Fischler, I.R. Klebanov and L. Susskind, *Schwarzschild black holes from matrix theory*, *Phys. Rev. Lett.* **80** (1998) 226 [hep-th/9709091].

[30] I.R. Klebanov and L. Susskind, *Schwarzschild black holes in various dimensions from matrix theory*, *Phys. Lett.* **B 416** (1998) 62 [hep-th/9709108];
G.T. Horowitz and E.J. Martinec, *Comments on black holes in matrix theory*, *Phys. Rev.* **D 57** (1998) 4935 [hep-th/9710217];
E. Halyo, *Six dimensional Schwarzschild black holes in M(atrix) theory*, hep-th/9709225;
M. Li, *Matrix Schwarzschild black holes in large-N limit*, *J. High Energy Phys.* **01** (1998) 009 [hep-th/9710226];
T. Banks, W. Fischler, I.R. Klebanov and L. Susskind, *Schwarzschild black holes in*

matrix theory, 2, *J. High Energy Phys.* **01** (1998) 008 [hep-th/9711005];
T. Banks, W. Fischler and I.R. Klebanov, *Evaporation of Schwarzschild black holes in matrix theory*, *Phys. Lett.* **B 423** (1998) 54 [hep-th/9712236];
S. Kalyana Rama, *Matrix theory description of Schwarzschild black holes in the regime* $n \gg s$, *Phys. Rev.* **D 59** (1999) 024006 [hep-th/9806225].

[31] S.R. Das, S.D. Mathur, S. Kalyana Rama and P. Ramadevi, *Boosts, Schwarzschild black holes and absorption cross-sections in M-theory*, *Nucl. Phys.* **B 527** (1998) 187 [hep-th/9711003].

[32] F. Englert and E. Rabinovici, *Statistical entropy of schwarzschild black holes*, *Phys. Lett.* **B 426** (1998) 269 [hep-th/9801048];
R. Argurio, F. Englert and L. Houart, *Statistical entropy of the four dimensional Schwarzschild black hole*, *Phys. Lett.* **B 426** (1998) 275 [hep-th/9801053].

[33] E. Witten, *Noncommutative geometry and string field theory*, *Nucl. Phys.* **B 268** (1986) 253.

[34] V.A. Kostelecky and S. Samuel, *The static tachyon potential in the open bosonic string theory*, *Phys. Lett.* **B 207** (1988) 169; *On a nonperturbative vacuum for the open bosonic string*, *Nucl. Phys.* **B 336** (1990) 263;
A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [hep-th/9912249];
N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [hep-th/0002211];
N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, *Nucl. Phys.* **B 583** (2000) 105 [hep-th/0002237].

[35] J.R. David, U(1) *gauge invariance from open string field theory*, *J. High Energy Phys.* **10** (2000) 017 [hep-th/0005085];
W. Taylor, *Mass generation from tachyon condensation for vector fields on D-branes*, *J. High Energy Phys.* **08** (2000) 038 [hep-th/0008033].

[36] U.H. Danielsson, A. Guijosa, M. Kruczenski and B. Sundborg, *D3-brane holography*, *J. High Energy Phys.* **05** (2000) 028 [hep-th/0004187].

[37] R. Gopakumar, S. Minwalla and A. Strominger, *Noncommutative solitons*, *J. High Energy Phys.* **05** (2000) 020 [hep-th/0003160].

[38] J.A. Harvey, *Komaba lectures on noncommutative solitons and D-branes*, hep-th/0102076.

[39] J.A. Harvey, D. Kutasov and E.J. Martinec, *On the relevance of tachyons*, hep-th/0003101;
A.A. Tseytlin, *Sigma model approach to string theory effective actions with tachyons*, *J. Math. Phys.* **42** (2001) 2854 [hep-th/0011033].

[40] E. Witten, *On background independent open string field theory*, *Phys. Rev.* **D 46** (1992) 5467 [hep-th/9208027]; *Some computations in background independent off-shell string theory*, *Phys. Rev.* **D 47** (1993) 3405 [hep-th/9210065];
K. Li and E. Witten, *Role of short distance behavior in off-shell open string field theory*, *Phys. Rev.* **D 48** (1993) 853 [hep-th/9303067].

[41] S.L. Shatashvili, *Comment on the background independent open string theory*, *Phys. Lett.* **B 311** (1993) 83 [hep-th/9303143]; *On the problems with background independence in string theory*, hep-th/9311177.

[42] J.A. Minahan and B. Zwiebach, *Field theory models for tachyon and gauge field string dynamics*, *J. High Energy Phys.* **09** (2000) 029 [hep-th/0008231].

[43] A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, *J. High Energy Phys.* **10** (2000) 034 [hep-th/0009103].

[44] D. Kutasov, M. Mariño and G.W. Moore, *Some exact results on tachyon condensation in string field theory*, *J. High Energy Phys.* **10** (2000) 045 [hep-th/0009148].

[45] P. Kraus and F. Larsen, *Boundary string field theory of the* $\mathrm{D}\bar{\mathrm{D}}$ *system*, *Phys. Rev.* **D 63** (2001) 106004 [hep-th/0012198];
T. Takayanagi, S. Terashima and T. Uesugi, *Brane-antibrane action from boundary string field theory*, *J. High Energy Phys.* **03** (2001) 019 [hep-th/0012210].

[46] M. Srednicki, *IIB or not IIB*, *J. High Energy Phys.* **08** (1998) 005 [hep-th/9807138];
E. Witten, *D-branes and K-theory*, *J. High Energy Phys.* **12** (1998) 019 [hep-th/9810188].

[47] A. Sen, *Supersymmetric world-volume action for non-BPS D-branes*, *J. High Energy Phys.* **10** (1999) 008 [hep-th/9909062].

[48] A.A. Gerasimov and S.L. Shatashvili, *Stringy Higgs mechanism and the fate of open strings*, *J. High Energy Phys.* **01** (2001) 019 [hep-th/0011009].

[49] J.A. Minahan and B. Zwiebach, *Gauge fields and fermions in tachyon effective field theories*, *J. High Energy Phys.* **02** (2001) 034 [hep-th/0011226].

[50] K. Hashimoto and S. Hirano, *Metamorphosis of tachyon profile in unstable D9-branes*, hep-th/0102174;
N.D. Lambert and I. Sachs, *On higher derivative terms in tachyon effective actions*, *J. High Energy Phys.* **06** (2001) 060 [hep-th/0104218].

[51] B. Craps, P. Kraus and F. Larsen, *Loop corrected tachyon condensation*, *J. High Energy Phys.* **06** (2001) 062 [hep-th/0105227].

[52] R. Rashkov, K.S. Viswanathan and Y. Yang, *Background independent open string field theory with constant B field on the annulus*, hep-th/0101207;
T. Suyama, *Tachyon condensation and spectrum of strings on D-branes*,

hep-th/0102192;
N.D. Lambert and I. Sachs, *String loop corrections to stable non-BPS branes*, *J. High Energy Phys.* **02** (2001) 018 [hep-th/0010045];
K.S. Viswanathan and Y. Yang, *Tachyon condensation and background independent superstring field theory*, hep-th/0104099;
M. Alishahiha, *One-loop correction of the tachyon action in boundary superstring field theory*, *Phys. Lett.* **B 510** (2001) 285 [hep-th/0104164];
K. Bardakci and A. Konechny, *Tachyon condensation in boundary string field theory at one loop*, hep-th/0105098;
G. Arutyunov, A. Pankiewicz and B. Stefanski Jr., *Boundary superstring field theory annulus partition function in the presence of tachyons*, *J. High Energy Phys.* **06** (2001) 049 [hep-th/0105238].

[53] W.-H. Huang, *Boundary string field theory approach to high temperature tachyon potential*, hep-th/0106002.

[54] M.C. B. Abdalla, A.L. Gadelha and I.V. Vancea, *Bosonic D-branes at finite temperature with an external field*, hep-th/0104068.

[55] K.R. Dienes, E. Dudas, T. Gherghetta and A. Riotto, *Cosmological phase transitions and radius stabilization in higher dimensions*, *Nucl. Phys.* **B 543** (1999) 387 [hep-ph/9809406].

[56] S.A. Abel, J.L.F. Barbon, I.I. Kogan and E. Rabinovici, *String thermodynamics in D-brane backgrounds*, *J. High Energy Phys.* **04** (1999) 015 [hep-th/9902058].

[57] J.J. Atick and E. Witten, *The hagedorn transition and the number of degrees of freedom of string theory*, *Nucl. Phys.* **B 310** (1988) 291;
M.J. Bowick and S.B. Giddings, *High temperature strings*, *Nucl. Phys.* **B 325** (1989) 631.

[58] S. Chaudhuri, *Deconfinement and the hagedorn transition in string theory*, *Phys. Rev. Lett.* **86** (2001) 1943 [hep-th/0008131]; *Finite temperature bosonic closed strings: thermal duality and the kt transition*, hep-th/0105110; *Finite temperature closed superstring theory: infrared stability and a minimum temperature*, hep-th/0105244;
I.I. Kogan, A. Kovner and M. Schvellinger, *Hagedorn transition, vortices and D0-branes: lessons from $2+1$ confining strings*, *J. High Energy Phys.* **07** (2001) 019 [hep-th/0103235];
J.L.F. Barbón and E. Rabinovici, *On the nature of the hagedorn transition in ncos systems*, *J. High Energy Phys.* **06** (2001) 029 [hep-th/0104169].

[59] I.R. Klebanov and A.A. Tseytlin, *Entropy of near-extremal black p-branes*, *Nucl. Phys.* **B 475** (1996) 164 [hep-th/9604089].

[60] J.M. Maldacena and L. Susskind, *D-branes and fat black holes*, *Nucl. Phys.* **B 475** (1996) 679 [hep-th/9604042].

[61] J. Maldacena and A. Strominger, *Black hole greybody factors and D-brane spectroscopy*, *Phys. Rev.* **D 55** (1997) 861 [hep-th/9609026].

[62] G. Siopsis, *Scalar absorption by spinning D3-branes*, *Phys. Rev.* **D 61** (2000) 024029 [hep-th/9909071];
S. Musiri and G. Siopsis, *Temperature of D3-branes off extremality*, *Phys. Lett.* **B 504** (2001) 314 [hep-th/0003284];
J.F. Vazquez-Poritz, *Absorption by nonextremal D3-branes*, hep-th/0007202.

[63] G. Policastro and A. Starinets, *On the absorption by near-extremal black branes*, *Nucl. Phys.* **B 610** (2001) 117 [hep-th/0104065].

[64] B. Zhou and C.-J. Zhu, *The complete black brane solutions in D-dimensional coupled gravity system*, hep-th/9905146;
P. Brax, G. Mandal and Y. Oz, *Supergravity description of non-BPS branes*, *Phys. Rev.* **D 63** (2001) 064008 [hep-th/0005242].

[65] G.T. Horowitz and K. Maeda, *Fate of the black string instability*, hep-th/0105111.

[66] S.S. Gubser and I. Mitra, *Instability of charged black holes in anti-de Sitter space*, hep-th/0009126;
S.S. Gubser and I. Mitra, *The evolution of unstable black holes in anti-de Sitter space*, *J. High Energy Phys.* **08** (2001) 018 [hep-th/0011127];
H.S. Reall, *Classical and thermodynamic stability of black branes*, *Phys. Rev.* **D 64** (2001) 044005 [hep-th/0104071].

[67] L. Susskind, *Holography in the flat space limit*, hep-th/9901079;
J. Polchinski, *S-matrices from AdS spacetime*, hep-th/9901076.